

GENERATION OF REGULAR MESHES FOR NUMERICAL WAVE MODELING

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Abstract: Numerical simulations in hydrodynamics based on the methods of finite differences require regular element meshes of the bathymetry. The approximation quality of these meshes as well as the shape of the elements strongly influence the quality of the results of numerical simulations especially in the area of wave forecast. In this paper an innovative approach is developed which facilitates the process of mesh generation based on initial measurement data. This approach is characterized by the approximation of bathymetries with b-spline surfaces. Regular meshes with different resolutions can be very easily generated based on these approximating free form surfaces. The efficiency and the effects of this approach is analysed within numerical wave simulations using the phase averaged wave model SWAN. These studies are carried out for the Weser River at the German North Sea coast.

Keywords: coastal engineering, numerical wave simulation, regular mesh generation, b-spline surfaces

1. INTRODUCTION

In the field of numerical simulations in coastal engineering these element meshes have to approximate the bathymetry with high accuracy. These bathymetries of a coast line or a course of a river is defined by three-dimensional measurement points gained by different depth sounding surveys, digital surface models or digitalisation of maps.

Figure 1 shows an overview of the estuary of the Weser River between the cities of Bremen and Bremerhaven located at the German North Sea coast. In this figure different measurement points of a typical bathymetry are shown. While the foreland and digitised data consists of 300.000 measurement points the amount of data for the navigable water goes beyond the scope of traditional meshing tools: the fairwaters 1999 and 2000 are characterized by 10.000.000 measurement points.

Because of the simplicity to generate rectangle elements usually orthogonal meshes as shown on the right hand side of Figure 2 are used for the numerical simulations, although the approximation quality is poor compared to adapted meshes. In this sense the terms rectangle and orthogonal are related to the xy-plane of the test area. The z-coordinate for depth information in all three-dimensional illustrations is scaled by the factor of 20. This orthogonal element mesh of the Weser Estuary is composed of 126*598 quadrilateral elements.

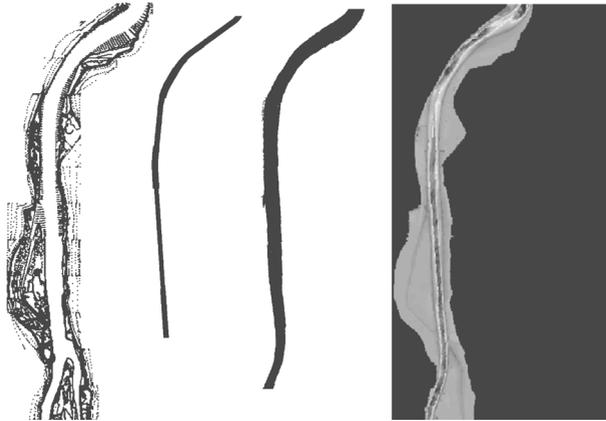


Figure 1: Measurement points of the Weser Estuary
(from left to right: foreland, navigable water 1999 and 2000, digitised data)

These three dimensional measurement points are approximated by free form surfaces based on the b-spline technique. In the presented approach, the b-spline technique is adapted to the specific requirements of the mesh generation process. A quadrilateral element mesh generated on the approximating b-spline surface with a 34×300 control point grid is shown in Figure 2 (right). The curvilinear mesh consists of 33×299 quadrilateral curvilinear elements.

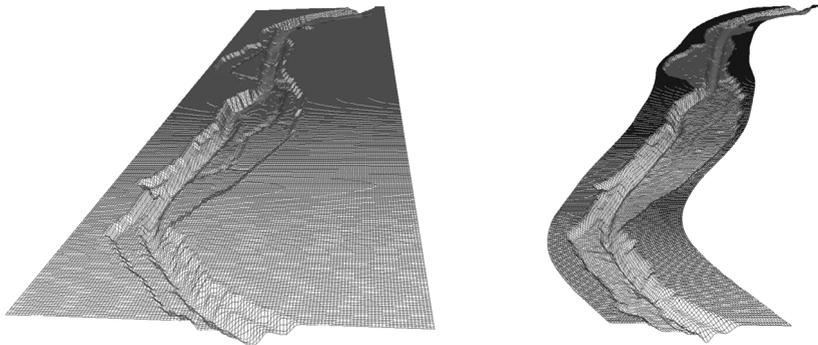


Figure 2: Orthogonal element mesh and b-spline surface and curvilinear element mesh

2. B-SPLINE TECHNIQUE

In computer aided geometric design and re-engineering of surfaces the technique of b-splines is widely used in order to describe the geometric shape of technical products. Bézier curves and surfaces fulfil often the requirement of generating smooth geometry. In the field of bathymetry approximation based on a large number of scattered data Bézier surfaces show the disadvantageous property of global modeling possibility. Therefore the concept of Bézier surfaces is generalized to the concept of segmented surfaces, which leads to the surface representation with b-spline technique [1]. A point on a b-spline surface is defined by

$$\mathbf{b}(u, v) = \sum_{i=0}^N \sum_{j=0}^M \mathbf{d}_{ij} N_i^K(u) N_j^L(v) \quad (1)$$

In equation (1) all expressions in bold face indicate a point in the three dimensional Euclidian space E^3 . On the left hand side of this equation $\mathbf{b}(u,v)$ denotes a point on the b-spline surface in dependence of the two parameters u and v . The first expression \mathbf{d}_{ij} in the double sum describes a regular grid of $N+1$ control points in u parameter direction and $M+1$ control points in v parameter direction. The shape functions in u and v parameter directions are called b-spline functions $N_i^K(u)$ and $N_j^L(v)$, where the upper indices K and L indicate the degree of the b-spline functions. In order to ensure the property of local modeling possibility the influence of the control points with respect to the shape of the surface has to be restricted to a specified parameter range. Therefore the b-spline functions of degree 0 are defined as follows

$$N_i^0(u) = \begin{cases} 1 & \text{for } u \in [u_i, u_{i+1}[\\ 0 & \text{else} \end{cases} \quad \text{for } i = 0, \dots, N + K \quad (2)$$

In equation (2) u_i and u_{i+1} denote the lower and upper bounds of the i^{th} parameter interval. All bounds of the parameter intervals are gathered in the knot vector \mathbf{u}

$$\mathbf{u} = [u_0, \dots, u_{N+K+1}]^T \quad (3)$$

The b-spline functions of higher degree are given with the recursive formula

$$N_i^r(u) = \frac{u - u_i}{u_{i+r} - u_i} N_i^{r-1}(u) + \frac{u_{i+r+1} - u}{u_{i+r+1} - u_{i+1}} N_{i+1}^{r-1}(u) \quad \text{for } \begin{matrix} r = 1, \dots, N + K \\ i = 0, \dots, N + K - r \end{matrix} \quad (4)$$

The b-spline function N_i^r of degree r is based on the b-spline functions N_i^{r-1} and N_{i+1}^{r-1} which ensures the important property of local modeling possibility

$$N_i^r(u) = 0 \quad \text{for } u \in \mathbb{R} \setminus]u_i, u_{i+r+1}[\quad (5)$$

The equation (1) has to be an affine combination which means that the factors of all control points for every parameter combination u and v have to summarize to 1. This requirement is only fulfilled within the interval

$$\sum_{i=0}^N N_i^K(u) = 1 \quad \text{for } u \in [u_K, u_{N+1}] \quad (6)$$

which leads to the restriction for the parameters $u \in [u_K, u_{N+1}]$ and $v \in [v_L, v_{M+1}]$ in definition (1).

3. B-SPLINE SURFACE EDITOR

Due to the different measurement point structures specific editor functionality is realised in order to generate very effectively a starting control point grid for the approximation b-spline surface. This starting grid has to fit to the course of the river or the coast line in the xy -plane. Figure 3 shows the editing process of a control point grid for a small detail of the Weser Estuary. As shown of the left hand side b-spline curves are used for the definition of the shore lines. Since the boundary curves are defined the control points are interpolated via bilinear blended Coons patches as shown on the right hand side.

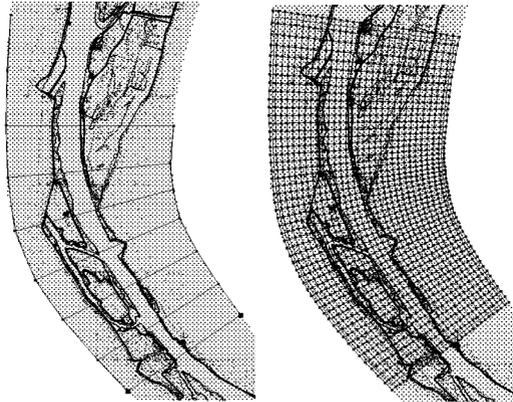


Figure 3: Editing process of approximating b-spline surfaces

4. APPROXIMATION OF THE BATHYMETRY AND MESH GENERATION

After the editing process the b-spline surface is generated in the plane $z = 0.0$ the realistic z -coordinates of the control points are still unknown. The application of equation (1) for every measurement point leads to an over-determined set of equations, which is to be solved with a QR decomposition method. In realistic cases this is a time and memory consuming process. A work around is the partition of the test area into several smaller b-spline surfaces [2].

In order to avoid these problems a high efficient iteration algorithm is developed [3]. For every control point \mathbf{b}_{ij} the corresponding close-by measurement points \mathbf{p}_m are selected. Based on these selected measurement points the mean values of their z -coordinates are calculated. Every mean value \mathbf{b}_{ij} has to fit to the z -coordinate of the b-spline surface at the parameters \bar{u}_i and \bar{v}_j of the corresponding de Boor point \mathbf{b}_{ij} :

$$\mathbf{b}_{z_{ij}} = \mathbf{b}_z(\bar{u}_i, \bar{v}_j) \quad (7)$$

The z -coordinates of all de Boor point $\mathbf{d}_{z_{ij}}^0$ are set to $\mathbf{b}_{z_{ij}}$ which leads to the deviation $\Delta \mathbf{d}_{z_{ij}}^0$ at the first iteration step

$$\mathbf{d}_{z_{ij}}^0 = \mathbf{b}_{z_{ij}} \Rightarrow \Delta \mathbf{d}_{z_{ij}}^0 = \mathbf{b}_z(\bar{u}_i, \bar{v}_j, \mathbf{d}_z^0) - \mathbf{b}_{z_{ij}} \quad (8)$$

The upper index indicates the iteration step. The z -coordinates $\mathbf{d}_{z_{ij}}$ of all de Boor points are gathered in the vector \mathbf{d}_z . These z -coordinates $\mathbf{d}_{z_{ij}}$ will be improved within the iteration:

$$\mathbf{d}_{z_{ij}}^k = \mathbf{d}_{z_{ij}}^{k-1} + \Delta \mathbf{d}_{z_{ij}}^{k-1} \text{ for } k > 0 \text{ with } \Delta \mathbf{d}_{z_{ij}}^k = \mathbf{b}_z(\bar{u}_i, \bar{v}_j, \mathbf{d}_z^k) - \mathbf{b}_{z_{ij}} \quad (9)$$

The iteration process will be continued until all deviations $\Delta \mathbf{d}_{z_{ij}}^k$ are small enough. This dragging algorithm drags the approximating surface towards the mean values \mathbf{b}_{ij} and is proved for the calculation of large-scale b-spline surfaces with several thousands of control points [3,4]. Since the approximating b-spline surface is based on a regular de Boor point grid it is very easy to generate a regular quadrilateral element mesh. The element nodes \mathbf{n} are generated in the uv -parameter space of the b-spline surface with equal parameters distances Δu and Δv .

5. APPLICATION IN NUMERICAL WAVE SIMULATION

The type and the resolution of the bathymetric mesh is a key factor in numerical simulation of waves. This holds especially for the numerical wave simulation in rivers because of their structured bathymetries as analysis with the third generation wave model SWAN [5] show. The basis of SWAN operated in the stationary mode is the action balance equation

$$\frac{\partial}{\partial x} c_x N + \frac{\partial}{\partial y} c_y N + \frac{\partial}{\partial \sigma} c_\sigma N + \frac{\partial}{\partial \theta} c_\theta N = \frac{S}{\sigma} \quad (10)$$

The first and the second term on the left hand side represent the propagation of action in geographical space (with propagation velocities c_x and c_y in x - and y -space). The third term describes the frequency shift due to variations in depths (with propagation velocity c_σ in σ -space). The fourth term models the depth-induced refraction (with propagation velocity c_θ in θ -space). The right hand side of the action balance equation is the source respectively sink term of energy density representing the effects of generation, dissipation and nonlinear wave-wave interactions. Equation (10) is discretized in σ -, θ -space as well as in x -, y -space using the bathymetric mesh constructed by b-splines. The standard physical processes incorporated in the source term were activated applying the standard parameter set of SWAN [6,7] with slight modifications based on a calibration with on-site measurements [8] and physical modeling [9].

6. MODELING APPROACH AND RESULTS

The numerical wave forecast in the Weser River was carried out for two grid types (rectangular resp. curvilinear) and three grid resolutions (63x299, 126x598, 189x897 resp. 33x299, 66x598, 99x897).

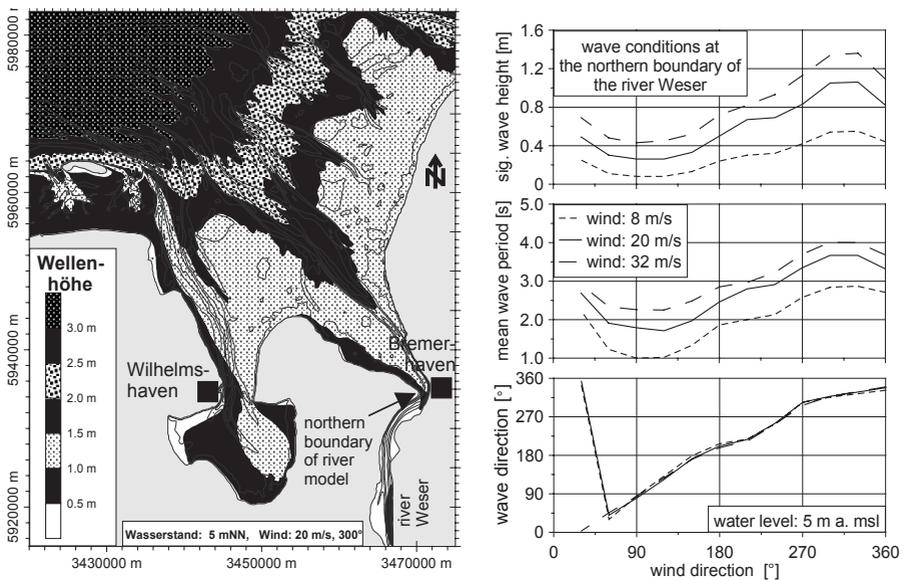


Figure 4: Wave field in the outer estuary and boundary conditions at the northern boundary of the Weser River

The incoming wave field at the northern boundary was derived from numerical simulations of wave propagation within the outer estuary [10]. Figure 4 gives an example of the wave field in the outer estuary and the incoming wave conditions. Water level and wind condition were set constant within the model area and varied from 0 m a. msl to 5 m a. msl respectively from 8 m/s to 32 m/s with directions from 0° to 360°. Figure 5 shows the wave field in a focus area within the Weser River for a coarse rectangular mesh and a fine curvilinear grid. Significant differences of wave parameter become obvious for the computation with different mesh resolutions and types.

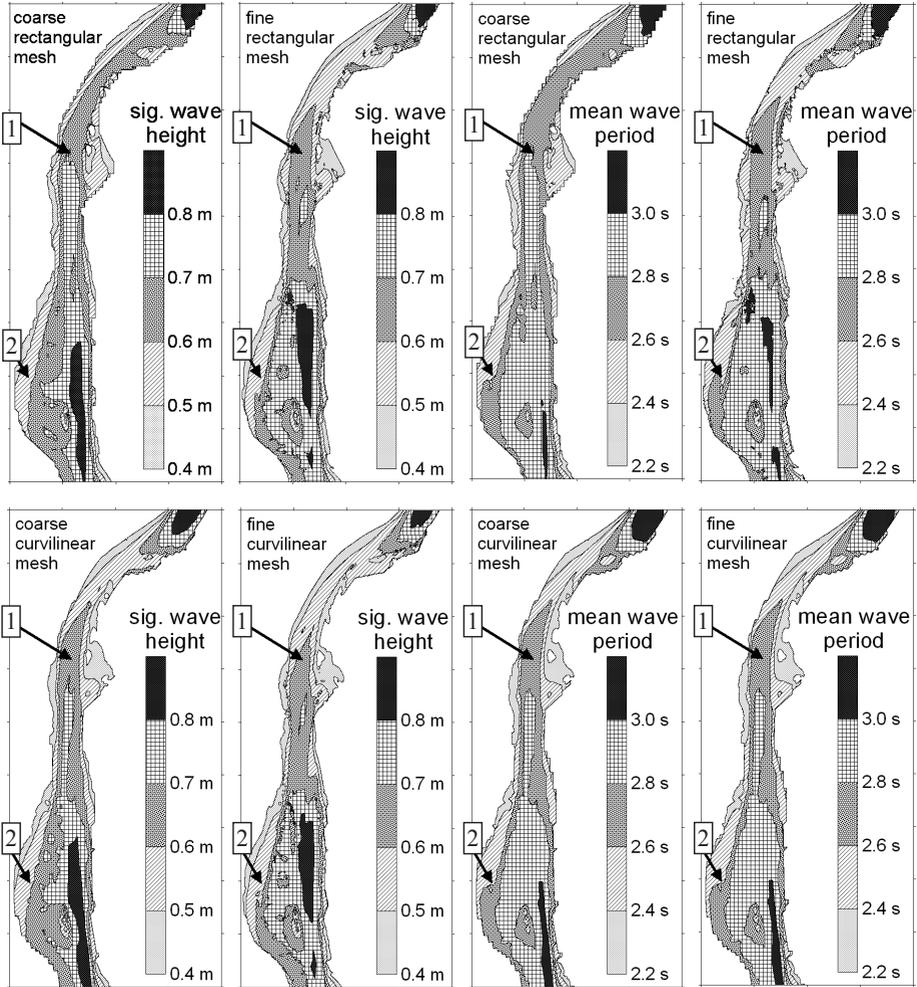


Figure 5: Wave field based on different types of meshes

Wave heights and periods are on average overestimated by SWAN simulations on coarse computational meshes. The same effect was observed [11] for short fetch conditions but not related to a coarse computational resolution of the bottom topography. The degree of over-

estimation is larger for rectangular meshes than for curvilinear meshes. However wave propagation in small side channels of the Weser River is calculated more accurately using curvilinear grids leading to larger wave heights and periods. For two locations, one in the main channel and the other in the side channel, a more detailed analysis is given in figure 6 and 7. All computational models lead to the similar directional dependence of wave parameters. At location 1 the wave heights differ up to 25 % while wave periods differ up to 11 %. At location 2 the differences are comparably smaller. Wave heights differ by a factor of 1.07 and wave periods by a factor of 1.05.

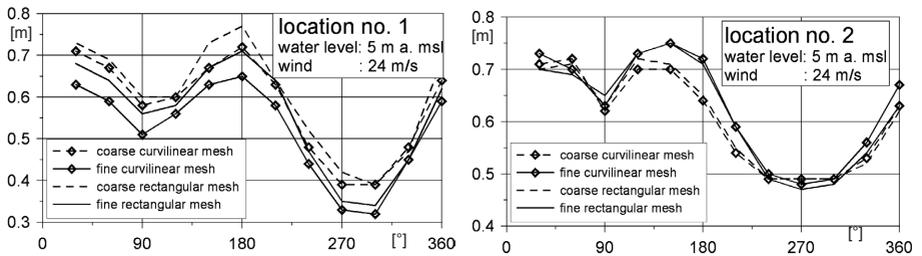


Figure 6: Significant wave height at location 1 and 2 in dependence of the wind direction

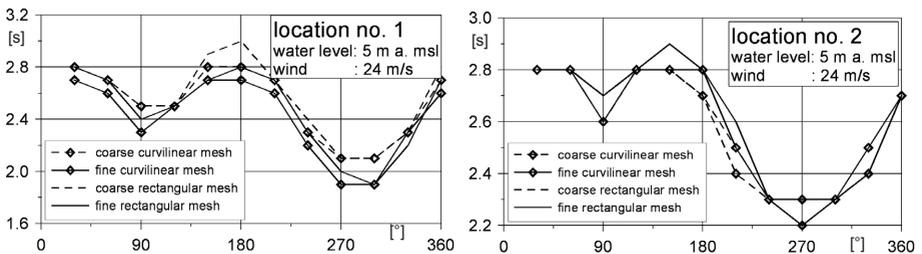


Figure 7: Mean wave period at location 1 and 2 in dependence of the wind direction

7. CONCLUSION

An accurate representation of the bathymetry is necessary for the numerical simulation of wave propagation and generation under short fetch conditions which are typical for rivers and estuaries. The presented approach has been proven to be suitable for practical applications. The best results are obtained by the wave model SWAN using high resolution curvilinear meshes requiring less computational time and memory. For the generation of curvilinear meshes the theory of b-spline approximation turned out to be very feasible because of its large data handling capacity and its in-build functionality of mesh refinement and adjustment.

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