On the Characteristics of Zero-crossing Waves in Model Wave Trains

K.-F. Daemrich
Franzius-Institute for Hydraulic, Waterways and Coastal Engineering, Leibniz University Hannover
daekf@fi.uni-hannover.de

S. Mai
BfG German Federal Institute of Hydrology
Mai@bafg.de

N. Kerpen
Franzius-Institute for Hydraulic, Waterways and Coastal Engineering, Leibniz University Hannover
kerpen@fi.uni-hannover.de

Abstract

The zero-crossing characteristics of wave trains, generated by inverse Fourier-transformation from wave spectra, depend on spectral shape, frequency spacing of the components (length of the time-series) and phase angles. The distribution of the wave heights is close to the Rayleigh distribution. In detail, variations in the distributions of high waves and of the highest waves occur. These variations increase with decreasing number of waves.

To highlight the consequences for model investigations, wave trains with different numbers of waves (30 to 1000) were generated and analysed with respect to the highest wave in the wave train and for the example of mean overtopping rates at a vertical wall.

Keywords: Model wave trains, Maximum wave heights, Overtopping, Data scatter, Rayleigh distribution

1. Introduction

Wave-trains in physical or numerical models are typically generated by inverse Fourier-transformation from the density distribution of a theoretical spectrum. The length of the time-series (number of waves N) is controlled by the frequency spacing of the Fourier components. The characteristics of zero-crossing wave heights and periods result from the spectral shape and the phase angels attributed to the components. These are usually taken as random. Different “seeds” of random phase angles result in different time-series. It is assumed that such wave-trains have the same characteristics as natural wave-trains. This is insofar appropriate, as the distributions of the zero-crossing wave heights at large are relatively good described by the Rayleigh distribution. Fig. 1 gives an example of the distribution of wave heights for a wave train with 1000 individual waves in comparison to the Rayleigh distribution. It should be mentioned that this is an example with good conformity. The range of possible deviation will be discussed later.

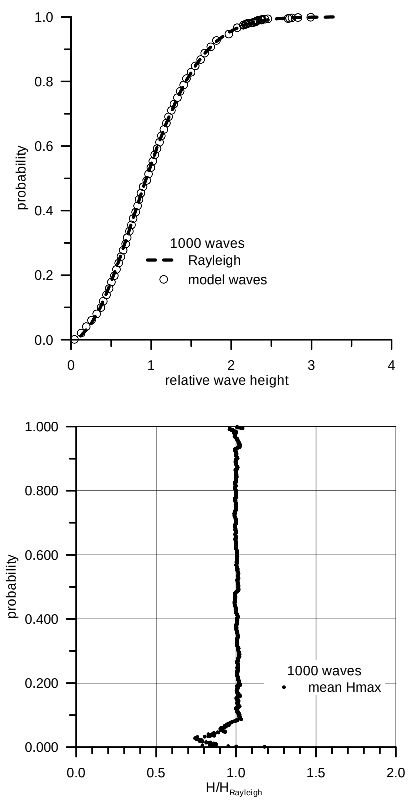

Fig. 1. Model wave heights in comparison to the Rayleigh distribution
top: comparison of distributions
bottom: relation model wave heights/Rayleigh heights

The influence of the spectral density distribution on the wave parameters and especially period distributions was investigated in an earlier paper by Daemrich et al.[1] and are not treated here.

Analysing such time-series in detail, we find variations in the distribution of the high waves and especially a variation of the maximum wave in the various wave-trains from same spectral density distributions. These variations in the distributions affect the model results, particularly in investigations of non-linear processes, as wave overtopping, maximum forces on structures etc. (however, would occur similarly in natural wave trains).

As the variations of the distributions and therewith the model test results can be very much dependent on the number of waves in the model wave-train, it should be known, how far results can be influenced by the number of waves. A general recommendation of 1000 waves does not meet the requirements in individual cases. Time-series with fewer waves might be sufficient or sometimes be indispensable, because of reflections in models or computing time.

In the following, wave-trains with various numbers of waves are investigated firstly with respect to the maximum wave.

The importance of the subject (unsteadiness in the distribution of the high waves) generally is demonstrated in a further chapter by the example of mean overtopping rates at vertical walls as a function of the freeboard R_c.

All wave-trains in this paper are generated from JONSWAP spectra by linear superposition (no bound lower and higher harmonics) with significant frequency domain parameters H_s = 1 m and T_p = 4 sec.

2. Maximum wave heights in time-series with various numbers of waves

For this investigation we have generated time-series by inverse Fourier transformation (linear superposition), varying the number of frequencies (block size) of the spectrum. The mean number of waves N ranged from about 30 to 1000 waves. For each typical length of the time-series, 64 realisations were produced, varying the random phases. In Table 1 some statistical information is assembled. Information on mean values of the mean number of waves N and the significant wave heights $H_{1/3}$ are also given.

Table 1. Basic statistics of wave height H_{max}
JONSWAP spectrum, H_{m0} = 1 m, T_p = 4 s

N	$H_{1/3}$ mean	H_{max} min	H_{max} mean	H_{max} max	CV H_{max}
30,2	0,942	1,05	1,326	2,08	0,137
61,3	0,958	1,15	1,398	1,80	0,102
124,6	0,960	1,26	1,520	1,92	0,099
251,2	0,963	1,36	1,611	2,07	0,090
504,6	0,963	1,44	1,717	2,10	0,073
1007,9	0,965	1,57	1,811	2,17	0,078

N = number of waves in a wave train
CV = Coefficient of variation

To give an impression on the related wave height distributions in the following Figures 2 and 3 plots of distributions from time-series with 30 and 1000 waves are presented.

In Fig. 2 (top) the distributions related to the maximum, mean and minimum H_{max} are plotted, In Fig. 2 (bottom) relative wave heights $H/H_{Rayleigh}$ are plotted to highlight the differences. In Fig. 3 a and b the relative wave heights for the time-series with 1000 waves are given.

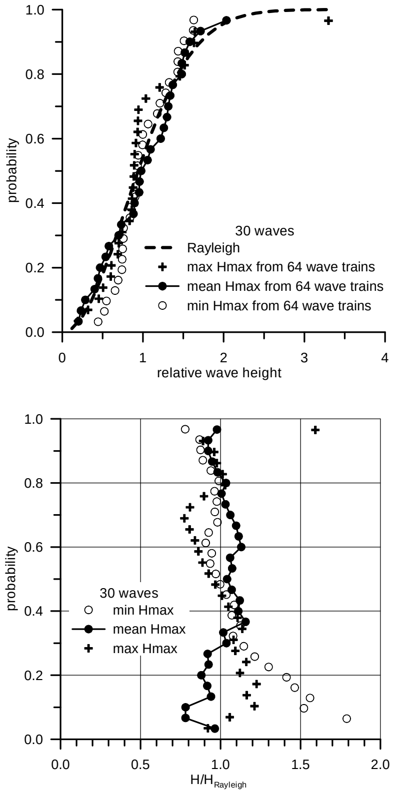

Fig. 2. Wave height distributions related to the max, mean and min H_{max} (30 waves)
top: absolute wave heights
bottom: relative wave heights $H/H_{Rayleigh}$

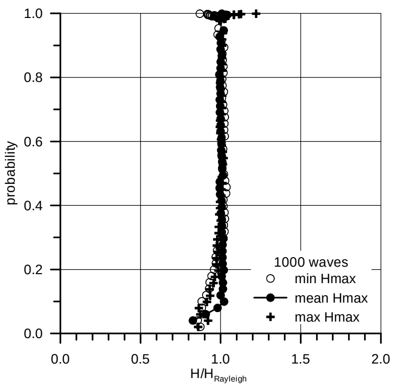

Fig. 3a. Relative wave heights $H/H_{Rayleigh}$ related to the max, mean and min H_{max} (1000 waves) (complete range of probability)

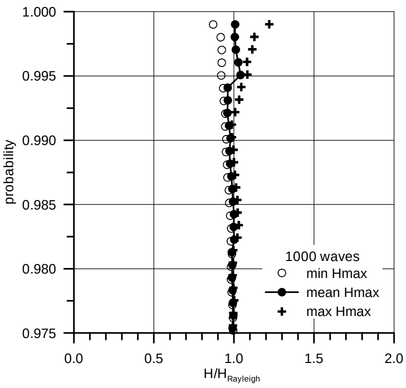

Fig. 3b. Relative wave heights $H/H_{Rayleigh}$ related to the max, mean and min H_{max} (1000 waves) (upper 2.5% range of probability)

Basically, from a given probability distribution of wave heights, a probability distribution for the maximum wave in a wave train with limited number of waves N can be derived (e.g. Massel and Sobey [3]). If the Rayleigh distribution is adopted as an approximation to the distribution of individual zero-crossing wave heights, the mean H_{max} as a function of the number of waves N is given by

$$\left(H_{max}/H_{1/3}\right)_{mean} \cong 0{,}706\left[\sqrt{\ln N} + \gamma/2\sqrt{\ln N}\right] \qquad (1)$$

with: $\gamma = 0.57722\ldots$ (Euler's constant)

The result of this theoretical approach is compared to the results from the data in Table 2.

Table 2: Comparison of results from data with Rayleigh theory

Number of waves N mean	mean $H_{max}/H_{1/3}$ data	mean $H_{max}/H_{1/3}$ theory
30,32	1,408	1,413
61,28	1,459	1,533
124,64	1,583	1,644
251,19	1,673	1,744
504,55	1,783	1,843
1007,86	1,877	1,935

For all time-series the relation $H_{max}/H_{1/3}$ from the data is less than the theoretical relation. This can be seen as a first hint, that the Rayleigh distribution overestimates high wave heights.

Using a normalized maximum wave height $R_{max} = H_{max}/H_{rms}$, the cumulate distribution function $F_{Rmax}(R_{max})$ for the maximum wave in a wave train with number of waves N is

$$F_{R\,max}\left(R_{max}\right) = \left[1 - \exp\left(-R_{max}{}^{2}\right)\right]^{N} \qquad (2)$$

(see also Massel and Sobey [3]). For the cases with about 30, 125 and 500 waves the theoretical cumulative distribution functions have been calculated and are compared to the data (Fig. 4). The curves in the plots are calculated with the mean number of waves, the dots are calculated with the corresponding number of waves of each sample.

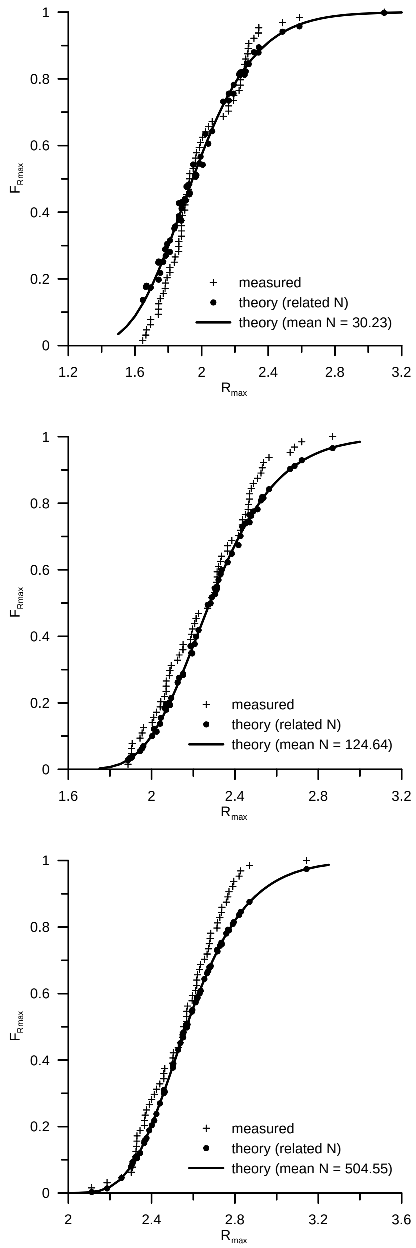

Fig. 4. Theoretical cumulative distribution function compared to the data (cases with 30, 125 and 500 waves)

It is obvious, that with increasing number of waves (and increasing mean H_{max}) the upper tail of the distribution is overestimated by the theoretical distribution function. This in addition indicates the trend of the Rayleigh distribution to overestimate high wave heights. From the trends of the measured cumulate distribution function $F_{Rmax}(R_{max})$ for the maximum waves, a modified probability distribution function of wave heights may be speculated.

3. Expected scatter in model tests for the example of overtopping rates at vertical walls

The importance of the subject (unsteadiness in the distribution of the high waves) generally and especially for hydraulic model tests is demonstrated by the example of mean overtopping rates at vertical walls as a function of the freeboard. The overtopping rates were calculated on the basis of the "probability calculation method" (Daemrich et al. [2]). Again, for each typical length 64 variations were investigated.

Fig 5 exemplifies the variation of overtopping rates for time-series with different numbers of waves in comparison to design functions in order to highlight also the problem of deriving design formulae from scattered data.

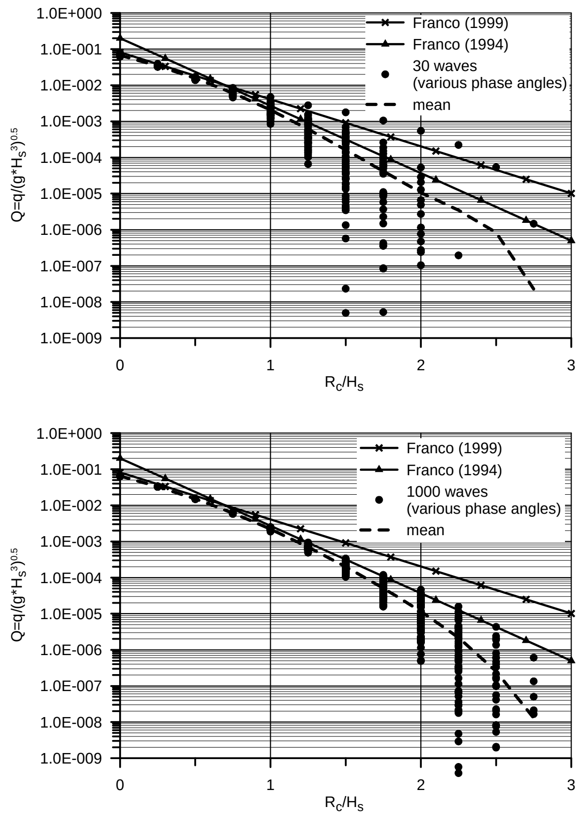

Fig. 5: Variation of mean overtopping rates from various time-series top: 30 waves; bottom: 1000 waves

As to be expected, the scatter of the overtopping rates is larger for the time-series with 30 waves. However, even with time-series of 1000 waves there is a considerable scatter for R_c/H_s > about 1.5.

To get an impression of the characteristics of the scatter in mean overtopping rates to be expected as a function of the length of the time series and the relative freeboard, mean values, standard deviations and relative standard deviations (coefficients of variation) are calculated from the results and plotted in Fig. 6 to 9.

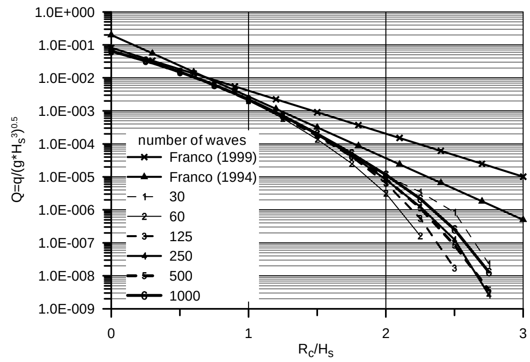

Fig. 6. Mean overtopping rates as a function of number of waves and relative freeboard

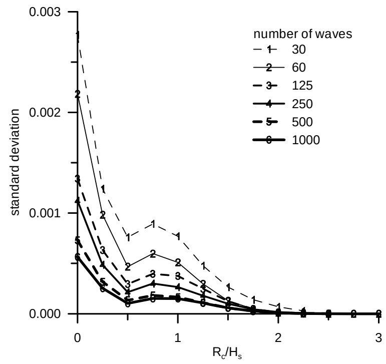

Fig. 7. Standard deviations for various numbers of waves and relative freeboards

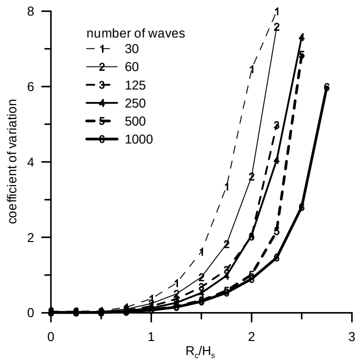

Fig. 8. Coefficients of variation for various numbers of waves and relative freeboards

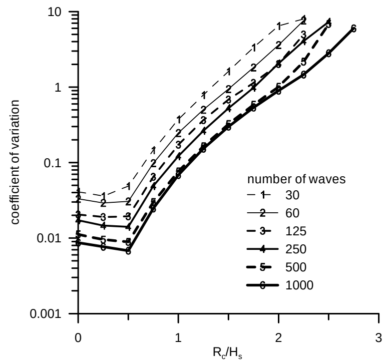

Fig. 9. Coefficients of variation for various numbers of waves and relative freeboards (logarithmic vertical axis)

Concerning the mean of the overtopping rates (Fig. 6), the tendency is expected to be, that the mean overtopping rates for higher freeboards increase with increasing length of the time-series (what seems reasonable). However, the results from the short wave time-series (with about 30 waves only) are distinctly higher around the relative freeboard R_c/H_s = 2.5. This reveals, that for short series even with 64 realisations, the mean may not be described sufficiently accurate.

The standard deviations (Fig. 7) also principally show the expected tendency of decreasing with increasing wave numbers of the time-series. The same holds for the coefficients of variation (relative standard deviations). Fig. 8 gives an impression of the increase of the relative scatter with increasing freeboard.

In the plot of the coefficients of variation with logarithmic vertical axis (Fig. 9) a discontinuity around R_c/H_s = 0.5 is seen, which also appears in the plot of the standard deviations (Fig. 7). At the time we have no explanation.

To highlight the dependency on the number of waves in a wave-train, the standard deviation for the example of a relative freeboard R_c/H_s = 1.5 is plotted in Fig.10. Besides the data as calculated, data related to the mean value of all data are given (mean values were slightly different for each set of number of waves), and also the theoretical course.

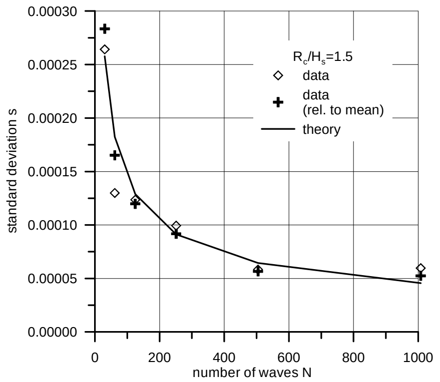

Fig. 10. Dependency of the standard deviation on the number of waves in a wave-train (example R_c/H_s = 1.5)

Based on such measured standard deviations the reliability of model test results with respect to the number of test runs n and lengths of wave trains N (N = number of waves in a wave-train) can be evaluated.

Here we assume the standard procedure for model tests (the result is the mean value $\bar{x}$ from a number n of test runs), and determine the confidence interval

$$\bar{x} \pm \frac{t}{\sqrt{n}} \cdot s$$

s = standard deviation
t = factor from Student's distribution

The term $\frac{t}{\sqrt{n}} \cdot \frac{s}{\bar{x}}$ is the relative deviation from $\bar{x}$. Examples for various lengths of wave-trains N and numbers of runs n are compiled in Table 3. For these examples, the standard deviation is taken from the theoretical values fitted to the data (see Fig. 10), and the t-factors for the 95% confidence will be used.

The results in lines 1 to 4 show the decrease of the relative deviation from the mean, when for a constant total number of waves N·n (here e.g. 2000) N is reduced and n is increased similarly. The results in lines 5 to 8 give the necessary number of runs n to have a relative deviation from the mean less than 0.5.

Table 3: Examples of relative deviation from the mean as a function of number of waves N and number of runs n

N	s	n	t	$\frac{t}{\sqrt{n}} \cdot \frac{s}{\bar{x}}$
1000	0.260	2	12.710	2.34
500	0.368	4	3.181	0.59
250	0.520	8	2.365	0.44
125	0.735	16	2.131	0.39
1000	0.260	4	3.181	0.41
500	0.368	5	2.777	0.46
250	0.520	7	2.447	0.48
125	0.735	11	2.228	0.49

A confidence interval can only be determined with a minimum of two test runs. So, the sometimes used approach, to do just one test (even with a high number of waves) is a doubtful result.

5. Summary

Zero-crossing characteristics of model wave trains of various length (64 realisations each length) have been investigated with respect to the distributions of wave heights and the maximum wave heights H_{max}.

As to be expected, the scatter compared to the Rayleigh distribution (reference distribution) was wider in the short time-series. From the deviation of the mean values and the distributions of the H_{max} waves from the theoretical distributions it can be concluded, that the Rayleigh distribution overestimates high wave heights.

Based on overtopping calculations with the probability calculation method the consequence of the scatter in the distributions on model test results was demonstrated. From the measured standard deviations as function of length of the time-series (number of waves) and relative freeboard R_c/H_s, conclusions on the necessary length of time-series and number of measurements can be derived, to get confident test results.

6. References

[1] Daemrich, K.-F., S. Mai, N. Ohle, and E., Tautenhain, *Influence of Spectral Density Distribution on Wave Parameters and Simulation in the Time Domain*, 2nd Chinese-German Joint Symposium on Coastal and Ocean Engineering, Nanjing, China, 2004

[2] Daemrich, K.-F., G. Tack, C. Zimmermann, and H.-Y. Cheng, *Irregular Wave Overtopping Based on Regular Wave Tests*. Third Chinese-German Joint Symposium on Coastal and Ocean Engineering, Tainan, 2006

[3] Massel, S.R., and R.J. Sobey, *Distribution of the highest wave in a record, Coastal Engineering Journal,* Vol. 42, No. 2, 153-173, 2000